Momentum Equation Analysis of Converging Flows Within a Manhole Junction

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ABSTRACT
Water industries within the United Kingdom and worldwide are becoming increasingly interested in the development and utilisation of Real Time Monitoring/Control (RT-M/C) of distribution and waste water systems, and flooding issues. This paper describes the investigations undertaken by the author as part of the Pennine Water Group at the University of Sheffield, to develop a suite of algorithms, utilising quasi-steady state methodology for use within specific problematic RT-M/C of wastewater system scenarios, such as a manhole junction. The algorithms were developed by utilising, a novel laboratory scale (1:10) prototype research apparatus, in the form of a simple manhole junction, as shown in Figure 1. In addition to physical laboratory testing, the research also utilised Computational Fluid Dynamics (CFD), of which no data is presented in this paper, and theoretical mathematical modelling.

Figure 1. Laboratory Schematic.

Overall the research centred on the development of a set of RT-M/C tools and rule/knowledge based solutions (the algorithms). To be utilised as a proactive approach in providing enhancement of current system understanding in terms of hydraulic functionality and pollution transport. Specifically in terms of flow reduction (backwater) and pressure loss behaviour due to converging flow patterns within the manhole junction. Additionally the research was used as a validation tool applied to existing mathematical RT-M/C modelling approaches.

KEYWORDS:
Flooding Impact, Momentum Equation, Real Time Monitoring/Control
INTRODUCTION
Combined sewer systems, which are widely used in the UK, undertake the dual roles of passing domestic and industrial wastewater to the treatment works and also routing stormwater runoff away from urban areas. Due to the potentially large volumes of runoff which may result from a storm, it is neither economical nor practical to design sewer systems to convey the entire volume to the treatment works. Furthermore it is not economical or practical to design treatment works to effectively treat these storm volumes or to store them for later treatment. Therefore under certain flow conditions (typically during heavy rainfall events), the maximum design (flow) of the combined sewer system (without additional storage facilities) is reached and exceeded, resulting in the discharge of effluent from the waste water system. Through designed outlets, such as Combined Sewer Overflows (CSO), to a receiving water course, or from a manhole structure into possibly the urban area. The discharging excess flows from the sewer, either into the river, or urban area is however a sensitive issue and is governed by various forms of legislation which aim to minimise the impact of such (pollution) discharges upon the environment. Gupta and Saul (1996) reported that, ‘many studies have identified the first flush phenomenon as being a relatively high load of pollutants in the initial phases of combined sewer flow. In (particular) system(s) without storage, this first flush of pollutants may be discharged from the system and result in the heavy pollution of the receiving water course (or urban area)’. Guidance for minimising pollution incidents have been provided for sewerage undertakers has been given by the Urban Pollution Management (UPM) manual, as a valuable tool developed by a collaboration of representatives from wastewater industry, research and regulatory bodies. It defines urban pollution management as:

“The management of wastewater discharges from sewer and sewerage treatment systems under wet weather conditions, such that the requirements of the receiving water are met in a cost effective way” (FWR 1998).

Typically sewer system modelling has centred on the use of either hydraulic (system) modelling tools such as Info/hydro-works, Mouse, 3D-Net etc, where the sewer network is modelled by use of the Bernoulli energy equation. Or alternatively where the flow in a (model/real) sewer network is largely modelled upon the equality of water stages model, employed at pre-defined node points (pipe junctions).

Equality of water stages model: \[
\begin{align*}
    h_u &= h_l \\
    h_d &= h_l
\end{align*}
\]

Where: \(h_u\), \(h_d\), and \(h_l\) refer to the water depth upstream, downstream and laterally to the junction. However studies of laboratory constructed systems have shown that the use of the equality of water stages model, can be subject to inherent, cumulative errors Through observed ‘losses’ within the system around manhole junctions (Howarth 1985). Where further analysis of flow behaviour particularly at combining channel junctions (where two streams combine to form a third) has been identified as, of considerable importance, (Abdalallah et al 2005). Additional concern in the use of such system models has also been raised with respect to the transportation of pollutants within the sewer system, particularly within the region of the first flush load (Gupta and Saul 1996). The quantification of flow behaviour around pipe junctions, observed under laboratory conditions has subsequently been undertaken by the use of (pipe) Froude number analysis by Hager and Gisonni (2005). However such analysis/modelling was concluded to be for the purposes of assessment only.
of the effects of sewer manholes on the performance of a sewer system, or as a complement to existing software packages. Further errors in flow behaviour around pipe junctions derived through numerical analysis with respect to existing mathematical models such as, Saint Venant. The Gurram, Hsu and Shabayek models have also highlighted the need to utilise models based upon the conservation of momentum as stated by Abdalallah et al (2005) and Kesserwani et al (2007).

The application of Real Time Control (RTC) to waste water systems is not in essence a new concept. It has been supported and promoted by the U.S. Environmental Protection Agency (EPA) since the 1970’s (Stinson 2002). However, Colas et al (2004) states that only a few communities (service providers) have implemented RTC, or even considered it as a viable technology option. However Saul 2007 proposes that future environmental regulation will require billions of pounds in investments to improve the operation of existing UK sewerage systems and to control the wet-weather pollution. Schutze et al (2002) posed the question of ‘why should we bother with RTC’, which in turn was provided with the following three valid answers:

- There is, or has recently been significant progress in measurement technology
- The consideration of water quality based objectives and the integrated approach to control open up new potential
- Methodologies and tools assisting in the development of control procedures have improved

The research presented in this paper details physical (laboratory) modelling approaches of flow, pressure and (soluble) pollution transport, utilising Rhodamine WT, alongside Computational Fluid Dynamics (CFD) modelling of flow and pressure loss, applied to a simple junction manhole, as shown in Figure 1. This led to further research into the application of theoretical mathematical modelling of flow characteristics utilising the conservation of momentum equation. This project utilised the three distinct, but interrelated themes as a viable alternative to current hydraulic modelling techniques that employ the equality of water stages model at sewer junctions. As this methodology, it is hypothesised and suggested in previous studied research, is subject to accumulative errors due to particular hydraulic behaviours caused by backwater effects. This in turn, it is again hypothesised, increases the unknown effects of flood risk, through the discharge of polluted sewer flows and its associated management problems.

**METHODS**

The laboratory system constructed at the University of Sheffield, which was used to measure the hydraulic behaviour locally within and due to the effects of a simple waste water system manhole junction, under surcharged conditions.

Briefly, the parameters that were measured by the laboratory system, during each laboratory tests included:

- Flow (volumetric $\text{Ls}^{-1}$), as the basis of all laboratory tests undertaken, in that flow would be used as a marker to ensure correct hydraulic conditions within the laboratory system (turbulent flow) were being met.
- Pressure in milli-bars (mb).
- Soluble dye concentration in parts per million (ppm).
- Surcharge levels within the manhole.
Each of these parameters was measured with the use of factory/laboratory calibrated instrumentation. The laboratory (hydraulic) tests used a series of different flow rates magnitudes in the two inflow pipes to the manhole. In one pipe a constant flowrate magnitude was established and this was fixed over the duration of the test, termed the Base flow, $Q_{\text{BASE}}$. The flowrate magnitude in the second pipe was gradually increased in a series of steady state increasing volumetric steps, hereafter known as $Q_{\text{STORM}}$. For example in a particular laboratory test, the flow magnitude in leg A of the laboratory apparatus was set at a single constant volumetric flow rate of magnitude of 0.5 $\text{ls}^{-1}$, to represent a base/dry weather flow, ($Q_{\text{BASE}}$), whilst in leg B the flowrate was simultaneously increased in a series of steady state steps, as shown in Figure 2, where the flow rates, increased in the range from 0, 0.5......5 $\text{ls}^{-1}$ in 0.25 $\text{ls}^{-1}$ increments to give a storm flow, $Q_{\text{STORM}}$.

![Figure 2. $Q_{\text{STORM}}$ Flow Pattern.](image)

For each different flowrate the hydraulic conditions were allowed to settle for a period of 5 minutes. This time period of 5 minutes allowed a sufficient, as hypothesised by the author, settling period of the magnitude of $Q_{\text{STORM}}$ to overcome early flowrate instability, experienced immediately after a flowrate change as a result of flow control valve movement/changes increasing localised flowrate turbulence. In addition to the parameters stated above, the influence on the hydraulic behaviour alone on $Q_{\text{BASE}}$ of the supply head was also investigated. Each laboratory test was repeated a minimum of 5 times, for the purpose of determining the mean magnitude of each parameter measured. Each test would then have data for each parameter measured averaged according to each of the increasing flow steps, as shown in Figure 2 to give a single point value relating to the settling of each flow step. Initial laboratory tests showed there was little difference between the results of the laboratory tests undertaken at similar magnitudes, therefore it was hypothesised that the series of laboratory results for each magnitude of $Q_{\text{BASE}}$ utilised could be grouped further to provide a single test output trend for each magnitude of $Q_{\text{BASE}}$, with respect to flow characteristics, and in particular, backwater effects, pressure losses across the manhole, measured surcharge within the manhole and the transport of a soluble pollutant. This is shown in Figure 3 for backwater effects both with and without flooding occurring from the manhole.
Figure 3. Mean $Q_{\text{BASE}} = 0.5, 0.6...1.0$ l/s Flow Reduction Comparative Plot.

Figure 3 shows that in general as the magnitude of $Q_{\text{BASE}}$ increases so does the total reduction in $Q_{\text{BASE}}$ upon convergence with an increasing magnitude of $Q_{\text{STORM}}$, particularly up to the point at which flooding from the manhole was imminent. This is detailed by the addition of a series of individual 4th order polynomial trendlines, which could be used to model the flow reduction effects to a high degree of accuracy. This accuracy was assessed by consideration of the values for $R^2$ greater than 0.99, associated with each derived polynomial.

Figure 4 shows the measured pressure loss characteristics across the manhole in direct relation to the backwater (flow reduction) trends as presented in Figure 3, to the point at which flooding occurred from the manhole.

Figure 4. $Q_{\text{BASE}} = 0.5, 0.6...1.0$ l/s Mean Pressure Loss Across Manhole Comparative Plot.

Figure 4 shows, in relation to the flow reduction trends presented in Figure 3 that as the magnitude of $Q_{\text{BASE}}$ increases so does the ‘reaction’ upon convergence with $Q_{\text{STORM}}$, in the form of increased pressure loss across the manhole. These findings generally support the study undertaken by Howarth (1985), who stated that head loss was not a constant proportion of the downstream velocity head, but varied with the flowrate and hence velocity. This led to
the hypothesis by the authors that any subsequent modelling strategies developed to proactively determine backwater effects around the manhole junction would need to primarily based upon hydraulic (flowrate/velocity) effects.

**Theoretical Mathematical Modelling**

To model the laboratory system using the momentum equation it was initially hypothesised that the flow of water through the manhole could be modelled as flows through 2 converging pipes. This hypothesis is supported by Saiyudthong (2003), who proposed that the jet flow is generally contained, and not dispersed in manholes with benching structures present. The flow of water in leg A of the laboratory system was modelled through an angle of 30° as represented in Figure 5, with the additional general assumptions that the jets that issued from the pipes would have the following characteristics:

- Jet diameter = 50 mm
- Gravity effects are nil because the pipes are horizontal and there is no weight component in this direction, hence no contribution to momentum balance
- Reynolds modelling and not Froude, as gravity was not being considered
- Analysis would be undertaken up to the point at which flooding would occur from the manhole

The laboratory manhole was constructed with concrete benching, to represent a ‘real world’ manhole, and hence it was assumed that this benching provided the boundaries of the control volume. Hence from Newton’s law

\[
\text{Force} = \text{Rate of change of momentum}
\]

Hence, taking the ‘x’ direction as parallel to the longitudinal aspect of the laboratory rig, as shown, gives Force (Newton’s) on the Fluid in the control volume in the ‘x’ direction (\(F_x\) for each flow) as equal to the rate of increase of ‘x’ momentum,

\[
F_x = (\rho \cdot Q) \cdot (u_2 \cdot \cos \theta - u_1) - ((p_1 \cdot A_1 - p_2 \cdot A_2) \cdot \cos \theta)
\]

**Equation 1.** Force On Fluid In X Direction (\(F_x\)).
Similarly for Force (Newton’s) on the Fluid in the control volume in the ‘y’ direction (F_y):

\[ F_y = \left( \rho \cdot Q \cdot (u_2 \cdot \sin \theta - 0) \right) + p_2 \cdot A_2 \cdot \sin \theta \]

**Equation 2.** Force On Fluid In Y Direction (F_y)

By applying velocities corresponding to the constant base flows and steady state increasing (flow) steps it was possible to generate a data table for all (theoretical) forces in the ‘x’ and ‘y’ planes for each flow rate. These calculated forces were then used to ascertain an independent Resultant Force (F_R) acting upon the Fluid by resolving a simple force diagram generated from the forces of the two converging fluids and solving Equation 3:

\[ F_R = \sqrt{F_x^2 + F_y^2} \]

**Equation 3.** Resultant Force (F_R)

Which acted in the direction (F_Rd) (to the ‘x’ axis ‘datum’), given by:

\[ F_{Rd} = \arctan \left( \frac{F_x}{F_y} \right) \]

**Equation 4.** Direction Of Resultant Force (F_DIR).

It can be seen from Figure 6 that by plotting the calculated value for F_R against the laboratory measured total velocity in the system, as measured in the downstream leg of the junction, yields a strong linear relationship between the two parameters. This lead to the hypothesis that a methodology may be formed where the magnitude of the resultant force may be accurately calculated from the total velocity at the downstream end of the manhole.

**Figure 6.** Calculated Resultant Force v’s Total System Velocity
Similarly by plotting the measured base flowrate reduction against the calculated resultant force it can be seen from Figure 7 that it is possible to accurately model the reduction in $Q_{\text{BASE}}$ by the use of a single $2^{\text{nd}}$ order polynomial function.

\[
\text{Theoretical Base Flowrate} = -0.1853\text{Res}^2 + 0.5031\text{Res} + 99.693
\]

\[R^2 = 0.9991\]


\[\text{Base Flowrate Reduction} \text{ v}'s \text{ Resultant Force}\]

\[\text{Figure 7. Base Flowrate Reduction v's Resultant Force}\]

This, it was hypothesised would form the methodology for a subsequent modelling strategy for proactively determining base flowrate reduction upon convergence with a storm flowrate.

**RESULTS AND DISCUSSION**

The application of the theoretical model methodology presented in the previous section, in near real time to a repeat set of laboratory tests of the form discussed earlier to investigate backwater effects only is presented here. During these repeat set of laboratory tests the methodology, as developed utilising quasi-steady state methodology, was applied to raw data, at 1 second intervals. The difference (error) between the laboratory measured and theoretically predicted flowrate reduction is plotted as an absolute error (percentage) magnitude.

\[\text{Test Condition A1, A2, A3, A4, A5, A6} \text{ Absolute Theoretical Model Error Comparison}\]

\[\text{Figure 8. Tests In Leg A Model Application Comparison (Moving Averages)}\]
As can be seen from Figures 8 and 9, the magnitude of absolute error of the predicted (theoretical) backwater effect is consistently within the region of 3%. This is shown generally by the raw theoretical data, which is subject to signal ‘noise’ and the applied ‘smoothing’ 125 period moving average trendlines. The moving average trendlines were added for visualisation purposes only. It was hypothesised that the general 3% error magnitude represented the limits at which the derived modelling polynomial functions could be confidently be applied.

CONCLUSIONS

Laboratory testing has showed that the hydraulic behaviour of two distinct converging flow patterns (Q_{BASE}, Q_{STORM}) could be monitored and quantified, in terms of backwater effects to a high degree of accuracy.

Pressure losses across the manhole structure were quantified according to the measured backwater effect, and subsequently, accurately described by mid-order polynomial functions, up to the point at which flooding occurred from the manhole.

Theoretical mathematical modelling utilising the momentum equation yielded a model methodology based upon the calculation of a resultant force relative to two converging flow streams and then applying this resultant force to a second order polynomial function in order to accurately determine base flowrate reduction.

Application in near real time of the model methodology showed that there is a good degree of accuracy in the predictability of backwater (flow reduction) behaviour. This predictive accuracy of the methodology developed under quasi-steady state methodology, typically falls to within an absolute magnitude of 3%. The observed errors between experimental and theoretical flowrate reduction are however of low significance in terms of cause for concern, with regards to any subsequent model application, rather, that the magnitudes of the errors can be identified as, the limit to which the theoretical model can confidently be applied.

From the methodology derived, and its application against raw (laboratory) flow data, it has been shown that the Real Time collection of data and subsequent application of the model...
methodology, derived through the application of the momentum equation may form the foundation to a subsequent Decision Support Tool (DST).

It is stressed however that this research, whilst utilising a limited number of variables has provided initial steps to the development of a more complete DST.

REFERENCES
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FURTHER INFORMATION
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